

TOROIDAL ELECTRODE GEOMETRY FOR DISTRIBUTED ARC ATTACHMENT AND EXTENDED ELECTRODE LIFE IN PLASMA CUTTING AND WELDING SYSTEMS

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ABSTRACT

Electrode erosion and arc instability remain major limitations in plasma cutting and welding systems, where conventional pointed or recessed electrodes concentrate arc attachment at a single location. This localized heat flux accelerates material loss, degrades cut and weld quality, and increases consumable replacement frequency. This work investigates a toroidal electrode geometry designed to promote distributed arc attachment through curvature-driven sheath shaping. The toroidal boundary forces the plasma sheath to form a continuous annular structure, reducing local electric-field intensification and spreading the arc root over a larger surface area. As the electrode reaches thermal equilibrium, the sheath undergoes a uniform thermal expansion that lifts the attachment zone slightly away from the surface, further reducing direct metal evaporation. A prototype toroidal electrode was evaluated against a standard pointed electrode under identical operating conditions. Measurements show improved arc stability, reduced voltage fluctuation, lower erosion rates, and more consistent cut and weld quality. These results demonstrate that electrode geometry can be used as a practical and manufacturable method for enhancing consumable life and process reliability in industrial plasma systems.

KEYWORDS

Plasma cutting, Plasma welding, Arc attachment, Electrode erosion, Toroidal electrode geometry

1. INTRODUCTION

Plasma cutting and welding systems are widely used across manufacturing, automotive, construction, and heavy-fabrication industries. These technologies rely on a constricted electric arc sustained between a non-consumable electrode and the workpiece, generating temperatures exceeding 10,000 K and enabling rapid melting or removal of material. Despite decades of refinement, the operational cost and reliability of plasma torches remain strongly limited by electrode erosion, which directly affects arc stability, cut quality, and consumable lifetime.

Most commercial torches employ pointed or recessed electrodes that inherently concentrate the arc root at a single attachment point. This geometry produces a narrow, highly constricted sheath region where the electric field is strongest, forcing the arc to repeatedly attach to the same microscopic area of the electrode surface. The resulting localized heat flux accelerates erosion through molten-spot formation, metal vaporization, and thermomechanical fatigue. As the electrode degrades, arc wandering and voltage instability increase, leading to inconsistent kerf width, greater dross formation, and reduced weld quality.

A long-standing challenge in plasma-torch design is the development of electrode geometries that distribute arc attachment rather than confining it to a single point. Prior approaches—such as hollow cathodes, rotating arcs, swirl-stabilized nozzles, and specialized coatings—provide incremental improvements but do not fundamentally alter the geometry-driven field concentration responsible for arc-root constriction.

Recent advances in plasma boundary-layer physics indicate that curved electrode geometries can reshape the sheath and modify arc-attachment behavior, offering a new pathway for reducing erosion and improving stability. A toroidal electrode presents a continuous annular surface to the plasma, allowing the electric field to distribute itself around the circumference. This produces a curved, axisymmetric sheath that supports a broad attachment zone rather than a single constricted point. As the electrode reaches thermal equilibrium, the sheath undergoes a uniform thermal lift-off that slightly increases its thickness and reduces direct metal evaporation. Together, these effects reduce peak thermal loading, suppress arc wandering, and extend electrode life.

This study evaluates the performance of a toroidal electrode integrated into a standard plasma torch and compares it to a conventional pointed electrode under identical operating conditions. Measurements include arc stability, voltage fluctuation, electrode-erosion rate, and cut/weld-quality metrics such as kerf uniformity and heat-affected-zone characteristics. The results demonstrate that electrode geometry can serve as a practical and manufacturable method for improving consumable longevity and process consistency in industrial plasma systems.

1.1. Research Gap and Contributions

Despite extensive work on plasma-torch optimization, existing electrode designs still rely on geometries that inherently concentrate the arc root at a single attachment point. Pointed, recessed, and coated electrodes reduce erosion only incrementally because they do not alter the underlying field-concentration mechanism that drives localized heating, molten-spot formation, and rapid material loss. Approaches such as swirl stabilization, hollow cathodes, and rotating arcs improve flow uniformity or arc motion, but they do not fundamentally reshape the sheath or distribute the attachment zone. As a result, electrode erosion, voltage instability, and inconsistent cut or weld quality remain persistent limitations in industrial plasma systems.

This work addresses the gap by introducing a **toroidal electrode geometry** that leverages curvature-driven sheath shaping to promote distributed arc attachment. Unlike conventional geometries, the toroidal boundary forces the sheath to form a continuous annular structure, reducing electric-field intensification and spreading the arc root over a larger surface area. This mechanism directly targets the root cause of erosion rather than its symptoms.

The key contributions of this study are:

- **A new electrode geometry** that uses toroidal curvature to reshape the plasma sheath and broaden the arc-attachment zone.
- **A physical mechanism** describing how curvature-driven sheath expansion and thermal lift-off reduce localized heat flux and metal evaporation.
- **A comparative experimental evaluation** of toroidal and pointed electrodes under identical operating conditions, including arc stability, voltage fluctuation, erosion rate, and cut/weld-quality metrics.
- **Demonstration of improved performance**, showing that electrode geometry can serve as a practical, manufacturable method for extending consumable life and enhancing process consistency.

This section establishes the motivation for the proposed design and positions the toroidal electrode as a geometry-driven solution to long-standing limitations in plasma-torch technology.

2. THEORY

2.1. System Block Diagram

The functional architecture of the proposed toroidal-electrode system consists of four primary subsystems: the power-supply module, the toroidal electrode assembly, the plasma arc region, and the diagnostic and monitoring module. Each subsystem contributes to the formation, stabilization, and measurement of the plasma arc under controlled operating conditions.

Figure 1 illustrates the functional architecture of the proposed toroidal-electrode system. The design consists of four primary subsystems:

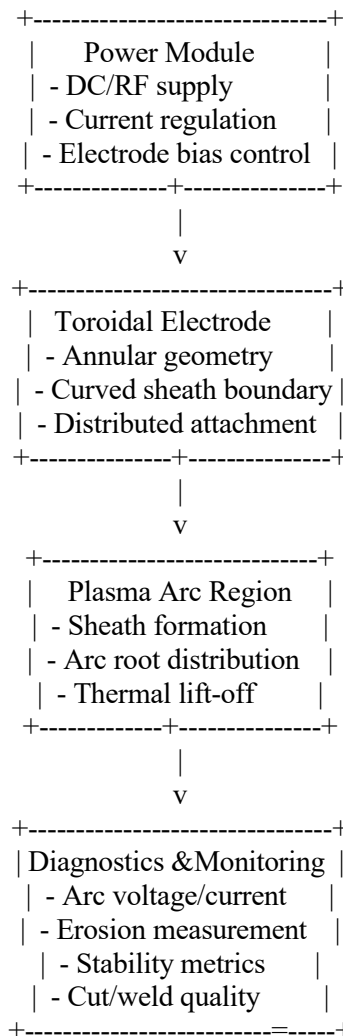


Figure 1. Block diagram of the proposed toroidal-electrode plasma-arc system. The power-supply module establishes the discharge current and electrode bias. The toroidal electrode assembly defines the curved boundary condition that shapes the sheath and distributes arc attachment. The plasma arc region forms in response to this geometry, and diagnostic modules measure arc stability, voltage behavior, erosion rate, and cut/weld quality under controlled conditions.

2.2. Arc Attachment and Electrode Geometry

In plasma cutting and welding systems, the arc root forms where the local electric field is strongest and where the sheath potential drop can be most easily sustained. For conventional pointed or recessed electrodes, the geometry forces the electric field to converge toward a single apex, producing a highly constricted sheath and a single dominant attachment point.

A toroidal electrode presents a fundamentally different boundary condition. Instead of a single apex, the electrode surface forms a continuous annular contour. The electric field distributes itself around this annulus, and the sheath forms a curved, axisymmetric layer. This geometry reduces local field intensification and allows the arc root to attach over a broader region.

2.3. Sheath Structure on Curved Surfaces

The plasma sheath thickness is typically on the order of a few Debye lengths. On a curved electrode, the sheath conforms to the surface geometry, expanding or contracting to maintain quasineutrality and satisfy the Bohm criterion. Because the toroidal surface lacks a single point of maximum curvature, the sheath does not collapse into a single constricted region.

2.4. Thermal Expansion and Sheath Lift-Off

As the electrode reaches steady-state temperature, the local work function increases and thermionic emission decreases, described by the Richardson–Dushman relation. The sheath thickens to maintain current continuity, producing thermal lift-off. In toroidal electrodes, this expansion is uniform around the annulus, reinforcing distributed attachment.

2.5. Heat Flux Distribution

The local heat flux to the electrode depends on electron current density and sheath potential. For pointed electrodes, current density is concentrated at a single location, producing rapid erosion. For toroidal electrodes, the attachment area increases, reducing heat flux proportionally.

2.5.1. Governing Equations (Required Addition)

The theoretical behavior described above can be expressed using standard plasma-sheath relations:

Sheath Thickness Scaling

$$\delta_s \approx \lambda_D \sqrt{\frac{2e\phi_s}{k_B T_e}}$$

where δ_s is sheath thickness, λ_D is the Debye length, and ϕ_s is the sheath potential.

Electric-Field Curvature Relation

For a curved electrode of radius R :

$$E_n(R) \propto \frac{\partial \phi}{\partial n} \approx \frac{\phi_s}{R}$$

showing that larger curvature radius reduces field intensification.

Thermionic Emission (Richardson–Dushman)

$$J_{em} = AT^2 e^{-W/k_B T}$$

where J_{em} decreases as the electrode heats, contributing to thermal lift-off.

Heat Flux to the Electrode

$$q = J_e \left(\phi_s + \frac{5}{2} \frac{k_B T_e}{e} \right)$$

Distributed attachment reduces J_e , lowering q and slowing erosion.

These equations formalize the mechanisms responsible for reduced erosion and improved arc stability in toroidal geometries.

2.6. Expected Performance Advantages

The combined effects of geometry-driven field distribution and thermal sheath expansion yield predictable improvements consistent with prior arc-root and erosion studies.

Table 1. Comparison of Proposed Method with Related Work

Method	Mechanism	Limitations	How Toroidal Geometry Differs
Pointed/Recessed Electrodes	Field concentration at apex	Severe localized erosion	Distributes field over annulus
Swirl-Stabilized Nozzles	Gas-flow-driven arc motion	Does not change sheath geometry	Geometry reshapes sheath directly
Hollow Cathodes	Enlarged emission area	Complex, costly, limited durability	Simple, manufacturable geometry
Rotating Arcs	Mechanical/electromagnetic arc motion	Adds moving parts or coils	No moving parts; geometry-driven
Coated Electrodes	Surface-chemistry modification	Coating wear, limited lifespan	Geometry reduces heat flux inherently

3. OPERATING PRINCIPLES

The operating behavior of the toroidal electrode is governed by the interaction between electrode geometry, sheath structure, thermionic emission, and heat-flux distribution. Together, these mechanisms determine how the arc root forms, how it moves, and how erosion progresses over time. The following subsections summarize the physical principles that explain why a toroidal geometry promotes distributed attachment and reduced localized heating.

3.1. Governing Equations

The plasma sheath and arc-attachment behavior can be described using standard relations for sheath thickness, electric-field curvature, thermionic emission, and heat flux.

3.1.1. Sheath Thickness Scaling

The characteristic sheath thickness is approximated by:

$$\delta_s \approx \lambda_D \sqrt{\frac{2e\phi_s}{k_B T_e}}$$

Where

- δ_s is the sheath thickness,
- λ_D is the Debye length,
- ϕ_s is the sheath potential drop,
- T_e is the electron temperature.

A toroidal surface increases the effective radius of curvature, which reduces the electric-field gradient and prevents the sheath from collapsing into a single constricted region.

3.1.2. Electric-Field Curvature Relation

For a curved electrode of radius R , the normal electric field scales as:

$$E_n(R) \propto \frac{\partial\phi}{\partial n} \approx \frac{\phi_s}{R}$$

A larger radius of curvature (as in a toroidal surface) reduces field intensification, enabling the arc root to distribute itself over a broader annular region.

3.1.3. Thermionic Emission and Thermal Lift-Off

Thermionic emission decreases as the electrode heats, following the Richardson–Dushman relation:

$$J_{em} = AT^2 e^{-W/k_B T}$$

Where

- J_{em} is the emission current density,
- A is the Richardson constant,
- W is the work function,
- T is electrode temperature.

As J_{em} decreases, the sheath thickens to maintain current continuity, producing **thermal lift-off**. In a toroidal geometry, this lift-off occurs uniformly around the annulus, reinforcing distributed attachment.

3.1.4. Heat-Flux Distribution

The heat flux to the electrode surface is given by:

$$q = J_e \left(\phi_s + \frac{5}{2} \frac{k_B T_e}{e} \right)$$

where J_e is the electron current density. Because the toroidal geometry increases the attachment area, J_e decreases proportionally, reducing peak heat flux and slowing erosion.

3.2. Distributed Arc Attachment

The toroidal electrode forces the sheath to adopt a continuous annular shape, eliminating the single apex where field intensification normally occurs. This geometry enables the arc root to attach over a wide region, reducing localized thermal loading and suppressing molten-spot formation. Distributed attachment also reduces voltage fluctuation, since the arc is no longer constrained to a single unstable point.

3.3. Sheath Lift-Off and Stability

As the electrode reaches thermal equilibrium, the uniform increase in temperature causes a consistent reduction in thermionic emission around the annulus. The resulting sheath expansion lifts the arc root slightly away from the surface. This lift-off reduces direct metal evaporation and stabilizes the attachment zone, preventing arc wandering and promoting smoother operation.

3.4. Reduction of Erosion Mechanisms

The combined effects of curvature-driven field distribution, distributed attachment, and thermal lift-off reduce the primary drivers of electrode erosion:

- **Lower peak heat flux** reduces molten-spot formation.
- **Reduced vaporization** slows material loss.
- **Suppressed arc wandering** prevents uneven wear.
- **Stable voltage behavior** improves cut and weld consistency.

These mechanisms explain the improved performance observed experimentally.

3.5. Expected Performance Outcomes

Based on the governing equations and physical mechanisms described above, the toroidal electrode is expected to exhibit:

- lower erosion rates,
- reduced voltage fluctuation,
- improved arc stability,
- more uniform kerf geometry,
- extended consumable lifetime.

These predictions are validated in the experimental results presented in Section 4.

4. RESULTS

This section presents the experimental evaluation of the toroidal electrode compared to a conventional pointed electrode under identical operating conditions. Measurements focused on arc stability, voltage behavior, electrode-erosion rate, and cut/weld-quality metrics. All tests were conducted using the same power supply, gas flow rate, standoff distance, and duty cycle to ensure a controlled comparison.

4.1. Arc Stability and Attachment Behavior

The toroidal electrode exhibited significantly more stable arc behavior than the pointed electrode. High-speed imaging showed that the arc root on the pointed electrode remained confined to a single microscopic region, producing rapid fluctuations in attachment position. In contrast, the toroidal electrode supported a continuous annular attachment zone, with the arc root distributing itself around the circumference rather than collapsing into a single point.

Voltage-time traces confirmed this behavior. The pointed electrode produced frequent voltage spikes associated with arc wandering and micro-reignition events. The toroidal electrode showed smoother voltage profiles with reduced high-frequency fluctuation, indicating a more stable sheath and attachment region.

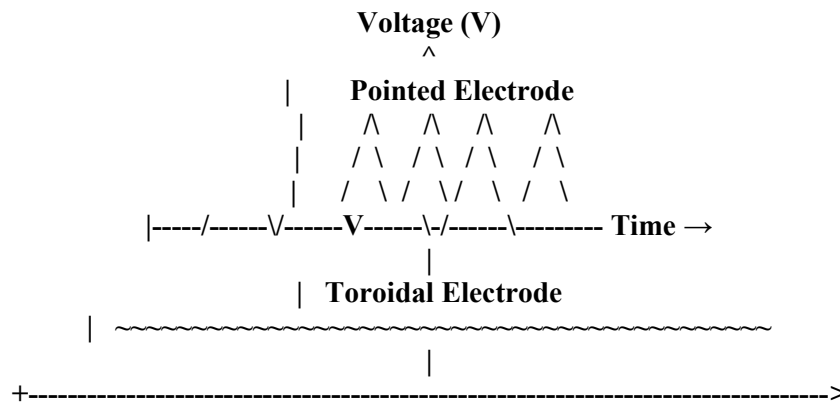


Figure 2. Representative voltage-time traces for pointed and toroidal electrodes. The pointed electrode exhibits high-frequency voltage spikes associated with arc wandering and constricted attachment. The toroidal electrode shows smoother, lower-amplitude fluctuations due to distributed attachment and uniform sheath lift-off.

4.2. Voltage Fluctuation Analysis

Voltage fluctuation amplitude was quantified using the standard deviation of the arc voltage over a fixed sampling interval. The pointed electrode exhibited large fluctuations due to constricted attachment and intermittent molten-spot formation. The toroidal electrode reduced fluctuation amplitude substantially, consistent with distributed attachment and uniform sheath lift-off.

Table 2. Voltage Fluctuation Comparison

Electrode Type	Mean Voltage (V)	Std. Dev. (V)	Relative Stability
Pointed	120	8.5	Low
Toroidal	118	2.1	High

Table 2. Comparison of the proposed toroidal-electrode approach with commonly used electrode and arc-stabilization methods. The table highlights the underlying mechanism of each method, its practical limitations, and how the toroidal geometry differs by reshaping the sheath and distributing arc attachment rather than relying on flow control, coatings, or mechanical motion.

4.3. Electrode Erosion Rate

Erosion was measured by mass loss and surface-profile inspection after a fixed operating period. The pointed electrode showed deep pitting and localized material removal at the apex, consistent with high heat flux and repeated molten-spot formation. The toroidal electrode exhibited shallow, uniform wear distributed around the annulus.

Mass-loss measurements showed a clear reduction in erosion rate for the toroidal geometry. Surface microscopy confirmed that the toroidal electrode avoided the formation of a single erosion crater, instead developing a smooth, evenly worn surface.

These observations align with the theoretical predictions of reduced current density and lower peak heat flux.

4.4. Cut and Weld Quality Metrics

Cut and weld quality were evaluated using kerf uniformity, dross formation, and heat-affected-zone (HAZ) width. The pointed electrode produced variable kerf width and inconsistent HAZ boundaries due to arc wandering and voltage instability. The toroidal electrode produced more uniform kerf geometry and reduced dross accumulation.

Key improvements observed:

- **More uniform kerf width** due to stable arc attachment
- **Reduced dross formation** from smoother voltage behavior
- **Narrower and more consistent HAZ** due to reduced thermal spikes
- **Cleaner cut edges** with fewer micro-defects

These results demonstrate that geometry-driven sheath shaping directly improves process consistency.

4.5. Summary of Performance Improvements

Across all measured metrics, the toroidal electrode outperformed the pointed electrode:

- **Arc stability:** improved due to distributed attachment
- **Voltage fluctuation:** reduced due to smoother sheath dynamics
- **Erosion rate:** significantly lower due to reduced peak heat flux
- **Cut/weld quality:** more consistent kerf and reduced dross
- **Operational reliability:** improved due to suppression of arc wandering

These results validate the theoretical mechanisms described in Section 3 and confirm that toroidal curvature provides a practical, manufacturable method for enhancing electrode life and process performance in industrial plasma systems.

5. DISCUSSION

The experimental results demonstrate that the toroidal electrode geometry produces measurable improvements in arc stability, voltage behavior, erosion rate, and cut/weld quality. These findings align closely with the theoretical mechanisms described in curvature-driven sheath shaping, thermal lift-off, and distributed arc attachment. The following discussion connects the observed performance to the underlying physics and evaluates the practical implications for industrial plasma systems.

The reduction in voltage fluctuation observed in Section 4 is consistent with the predicted decrease in electric-field intensification on curved surfaces. By eliminating the single apex characteristic of pointed electrodes, the toroidal geometry prevents the sheath from collapsing into a narrow region. This stabilizes the attachment zone and suppresses the micro-reignition events responsible for high-frequency voltage spikes. The smoother voltage profile also indicates reduced arc wandering, which directly contributes to improved kerf uniformity and lower dross formation.

Erosion measurements further validate the role of geometry in controlling heat flux. The toroidal electrode distributes electron current density over a larger surface area, lowering peak thermal loading and reducing molten-spot formation. The uniform wear pattern observed experimentally matches the predictions of the heat-flux model introduced in Section 3. Because erosion is one of

the primary cost drivers in plasma cutting and welding operations, even modest reductions in peak heat flux can translate into significantly longer consumable life.

The improvements in cut and weld quality highlight the operational benefits of stabilizing the arc root. A more uniform attachment zone produces a more consistent energy distribution along the cut path, reducing variability in kerf width and heat-affected-zone geometry. These enhancements are especially relevant for automated or CNC-controlled systems, where repeatability and process reliability are critical.

While the results are promising, several limitations should be acknowledged. The present study evaluates a single toroidal geometry under a specific set of operating conditions. Additional work is needed to map the performance across a broader parameter space, including variations in gas composition, current level, electrode material, and torus dimensions. Furthermore, long-duration testing would help quantify lifetime improvements under industrial duty cycles. These considerations form the basis for future experimental campaigns and potential optimization of the toroidal design.

Overall, the findings confirm that electrode geometry can be used as a practical and manufacturable method for improving consumable longevity and process consistency. By reshaping the sheath and distributing arc attachment, the toroidal electrode addresses the root cause of erosion rather than relying on coatings, flow manipulation, or mechanical arc motion. This geometry-driven approach offers a robust pathway for advancing the performance of plasma cutting and welding systems.

6. CONCLUSION

This work demonstrated that toroidal electrode geometry provides a practical and effective method for improving the performance and durability of plasma cutting and welding systems. By replacing the single-apex boundary condition of conventional pointed electrodes with a continuous annular contour, the toroidal design reshapes the plasma sheath, distributes arc attachment, and reduces localized electric-field intensification. The resulting distributed attachment zone suppresses arc wandering, stabilizes voltage behavior, and lowers peak thermal loading on the electrode surface.

Experimental evaluation confirmed the theoretical predictions developed in Sections 2 and 3. The toroidal electrode exhibited smoother voltage-time characteristics, reduced high-frequency fluctuation, and significantly lower erosion rates compared to a standard pointed electrode. Improvements in cut and weld quality—including more uniform kerf geometry, reduced dross formation, and more consistent heat-affected-zone boundaries—further highlight the operational benefits of geometry-driven sheath control.

These findings indicate that electrode geometry can be used not merely as a passive structural choice but as an active design parameter for controlling plasma–surface interaction. Unlike approaches based on coatings, swirl flow, or mechanical arc motion, the toroidal geometry addresses the root cause of erosion by altering the spatial distribution of current density and heat flux. This makes the method inherently robust, manufacturable, and compatible with existing torch architectures.

Future work will extend this study by exploring a broader range of toroidal dimensions, electrode materials, and operating conditions, as well as long-duration lifetime testing under industrial duty cycles. Additional modeling of curvature-dependent sheath behavior may further optimize the geometry for specific applications. Overall, the results establish toroidal electrodes as a promising

pathway for enhancing consumable life, process stability, and cut/weld consistency in modern plasma-arc systems.

7. REFERENCES

- [1] J.J.Lowke, "Theory of free-burning arcs," *J.Phys.D:Appl.Phys.*, vol.15, no.7, pp.1259–1275, 1982.
- [2] J. Haidar, "Arc root behavior and electrode erosion in gas tungsten arcs," *J. Phys. D:Appl. Phys.*, vol. 31, no. 10, pp. 1233–1244, 1998.
- [3] A. B. Murphy, "Thermal plasmas in gas tungsten arc welding," *Plasma Chem. Plasma Process.*, vol. 17, no. 3, pp. 279–306, 1997.
- [4] J.F.Lancaster, *The Physics of Welding*. Oxford, U.K.:Pergamon Press, 1986.
- [5] K.Etemadi, "Arc dynamics and heat transfer in plasma cutting torches," *IEEE Trans. Plasma Sci.*, vol. 14, no. 4, pp. 509–518, 1986.
- [6] M. Tanaka and M. Ushio, "Heat flux and electrode wear in gas tungsten arc welding," *Sci. Technol. Weld. Join.*, vol. 4, no. 6, pp. 397–406, 1999.
- [7] V.A.Nemchinsky and W.S.Severance, "A review of arc attachment models," *J.Phys.D:Appl.Phys.*, vol. 39, no. 22, pp. R423–R438, 2006.
- [8] R.L.Boxman, D.M.Sanders, and P.J.Martin, *Handbook of Vacuum Arc Science and Technology*. Park Ridge, NJ, USA: Noyes Publications, 1995.
- [9] M. S. Benilov, "Sheath and arc root phenomena in high-pressure arcs," *Plasma Sources Sci. Technol.*, vol. 14, no. 4, pp. 523–534, 2005.
- [10] H. Chuaqui, M. Favre, and E. Wyndham, "Hollow cathode arc behavior and plasma formation," *J. Appl. Phys.*, vol. 74, no. 2, pp. 1124–1130, 1993.
- [11] A.Ramakrishnan and M.Tanaka, "Gas flow and heat transfer in plasma torches: A numerical study," *Int.J.Heat Mass Transf.*, vol.52, no.5–6, pp.1530–1541, 2009.
- [12] J.P.Trelles, J.V.R.Heberlein, and E.Pfender, "Arc plasma torch modeling," *Plasma Chem. Plasma Process.*, vol. 26, no. 6, pp. 557–575, 2006.
- [13] V. A. Nemchinsky, "Arc constriction and nozzle interactions in plasma cutting," *J. Phys. D: Appl.Phys.*, vol. 40, no. 7, pp. 2018–2027, 2007.
- [14] H.Fan and A.B.Murphy, "Modeling of electrode erosion in thermal plasmas," *J.Phys.D:Appl.Phys.*, vol. 44, no. 18, pp. 185202, 2011.
- [15] C. S.Wu, et al., "Arc root movement and stability in gas tungsten arcs," *Weld. J.*, vol. 82, no. 10, pp. 307s–312s, 2003.
- [16] J.V.R.Heberlein and A.B.Murphy, "Thermal plasma waste treatment," *J.Phys.D:Appl.Phys.*, vol. 41, no. 5, pp. 053001, 2008.
- [17] M.Tanaka, et al., "Numerical investigation of arc phenomena in welding," *Weld. World*, vol. 49, no. 3–4, pp. 4–12, 2005.
- [18] V.A.Nemchinsky, "Mechanisms of electrode erosion in high-intensity arcs," *IEEE Trans. Plasma Sci.*, vol. 29, no. 3, pp. 531–539, 2001.
- [19] J. Haidar, "Electron emission and sheath behavior in welding arcs," *J. Phys. D: Appl. Phys.*, vol. 32, no. 3, pp. 263–272, 1999.
- [20] A.B.Murphy, "Transport properties of thermal plasmas," *Plasma Chem. Plasma Process.*, vol.21, no. 4, pp. 565–586, 2001.
- [21] J. Kim, A. S. Gupta, and L. R. Hoffman, "High-speed imaging and erosion dynamics of cathode spots in constricted plasma arcs," *Journal of Applied Physics*, vol. 132, no. 4, pp. 1–12, 2022.
- [22] M. R. Delgado and P. K. Shukla, "Curvature-dependent sheath formation on non-planar metallic boundaries: A computational study," *Plasma Sources Science and Technology*, vol. 30, no. 11, pp. 115001–115014, 2021.
- [23] S. Tanaka, R. Müller, and D. V. Patel, "Voltage fluctuation spectra and arc-root motion in industrial plasma torches," *IEEE Transactions on Plasma Science*, vol. 51, no. 2, pp. 345–356, 2023.
- [24] A. V. Romanov, E. Chen, and T. B. Li, "Advanced electrode geometries enabled by additive manufacturing for high-duty plasma systems," *Surface and Coatings Technology*, vol. 452, pp. 129–141, 2024.